APPENDIX I

EFFECTS OF IRREVERSIBLE PROCESSES ON THE CONSTITUTIVE RELATION

The constitutive relation (2.12) presented in Chapter II expresses the usual assumption of plasticity that stresses arise solely from elastic strains.[2,6] Because of high plastic strain-rates obtained in shock experiments, this assumption may be in error in that non-equilibrium contributions to the stress tensor could be significant. For this reason, irreversible effects are estimated in this appendix using a mathematical approach developed by Kluitenberg.[74]

For the case of small strains, Kluitenberg assumes that the strain tensor separates into elastic and inelastic contributions, as given in Chapter II. Furthermore, he assumes that total stress for a nonviscous medium is described by an equilibrium tensor, $S_{ij}^{(eq)}$, and an irreversible tensor, $S_{ij}^{(i)}$. These are defined as derivatives of entropy with respect to total and plastic strain components, respectively.

By using irreversible thermodynamics, he shows that the equation which relates stress and strain deviators is given in general as

$$K^{*}\eta^{(1)}S_{ij}^{(eq)} + \dot{S}_{ij}^{(eq)} = K(K^{*} - K)\eta^{(1)}T_{ij} + K\dot{T}_{ij} \quad , \qquad (I.1)$$

for an isotropic, nonviscous medium. In this equation, the notation of Chapter II has been used for strain tensors. K is an elastic modulus, the quantity $\eta^{(1)}$ is defined as

$$\dot{P}_{ij} = \eta^{(1)} S_{ij}^{(i)} \; , \tag{I.2}$$

and K^* is a constant relating to memory of the medium. Equation (I.1) is supplemented by equations of state relating the stress tensors to strain. For the linear theory, Kluitenberg expresses these as

$$S_{ij}^{(eq)} = KE_{ij} \; , \tag{I.3a}$$

$$S_{ij}^{(i)} = KT_{ij} - K^* P_{ij} \; . \tag{I.3b}$$

In order to estimate irreversible effects it is necessary to put Eq. (I.1) in a form similar to Eq. (2.12). If Eqs. (I.3a) and (I.3b) are combined and the result is substituted into (I.2), we obtain

$$\dot{P}_{ij} = \eta^{(1)} [S_{ij}^{(eq)} + P_{ij}(K - K^*)] \; . \tag{I.4}$$

Solving for $S_{ij}^{(eq)}$ and substituting this into Eq. (I.1) gives

$$\dot{S}_{ij}^{(eq)} = K\dot{T}_{ij} - K^* \dot{P}_{ij} + \eta^{(1)} [K(K^* - K)E_{ij} - (K^* - K)^2 P_{ij}] \; . \tag{I.5}$$

We note that this equation is similar to Eq. (2.12) if K is generalized to the elastic stiffness tensor. The main difference between the two calculations relates to the coefficients $\eta^{(1)}$ and K^*. Kluitenberg shows that if an elastic-plastic medium has no memory, $K=K^*$. In this case, Eq. (I.5) reduces identically to the constitutive relation used in Chapter II.

If the medium possesses memory, we can make a rough estimate of the magnitude of the term in brackets. Equations (I.2) and (I.3b) combine to give

$$\eta^{(1)} = \frac{\dot{P}_{ij}}{S_{ij}^{(eq)} + (K - K^*)P_{ij}} \; . \tag{I.6}$$